Synopsis

Supporting in-service professional development of mathematics teachers: The role of beliefs and knowledge

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1. Introduction

In India, various efforts have been undertaken to implement the new National Curriculum Framework 2005 (NCF 2005) that prioritizes learning with understanding and child-centered teaching (National Council of Educational Research & Training [NCERT], 2005). Teachers face challenges in implementing these goals and others such as relating school subjects with the daily lives of children. NCF 2005 has been criticized for being silent on how teachers are supposed to bring about the change in their classroom and for not addressing the much needed teacher development to support curriculum renewal (Batra, 2005). Efforts undertaken like changing textbooks and issuing directives to schools and teachers sidestep the issue of addressing beliefs and developing adequate knowledge amongst teachers, which is needed to realize the vision portrayed in the new curriculum framework. Although workshops have been conducted to "orient" the teachers to the new curriculum and textbooks, their impact on classroom teaching is doubtful.

NCF 2005 advocates a shift away from a textbook centered rote learning approach, to one that emphasizes the link between school learning and life outside school. It stresses that the knowledge that students bring to the classroom from their life outside, and the diversity of ability and ways of thinking within the classroom are resources for teaching and learning and not hindrances. Specifically with regard to mathematics, it gives precedence to the goal of mathematical thinking or mathematisation, rather than "knowing mathematics" as a set of rules and facts. The new curriculum, arguably expects from the teacher a deeper understanding of subject matter as well as the teaching learning process, rather than merely

adopting new techniques. Teachers in the elementary and middle grades are expected to not only make their students fluent in computational mathematics but also address process goals in the learning of mathematics, such as reasoning, using multiple ways to solve problems, justifying their solutions, making generalizations and conjectures, analyzing the mathematical work of others, etc. (NCERT, 2006). However, there have been few teacher professional development (TPD) programs in India, which have focused on the beliefs and knowledge required to facilitate this kind of teaching (Kumar, Dewan & Subramaniam, 2012). Studies elsewhere in the world have indicated that focus on change in teaching strategies without taking *teacher thinking* into consideration leads to teachers making superficial changes without any significant change in student learning opportunities (Cohen & Ball, 1990). It is therefore important to first understand beliefs and practices that are prevalent among teachers in order to support reform in teaching that is not superficial.

The present study is composed of four sub-studies presented in Chapters 4 to 7 of the thesis. Chapter 4 discusses the teachers' preferred practices as well as beliefs at the beginning of the study, which is the focus of sub-study 1. The findings of this sub-study serve as a background to the findings of the other sub-studies, in which teachers engaged in professional development activities. Sub-study 2 (Chapter 5) describes teachers' engagement in a professional development workshop by analyzing the tasks as well as interactions that occurred in the workshops. Sub-study 3 described in Chapter 6 is a case study of a teacher who participated in the orientation workshop and showed inclination to change her practices towards teaching mathematics with understanding. The sub-study highlights the challenges that arise when a teacher may agree with the philosophy of curriculum reform but still needs effort and relevant knowledge to engage students in developing an understanding of mathematics. Sub-study 4 highlights the role of developing specialized content knowledge amongst teachers by engaging in topic-focused professional development on the topic of integers. Teachers developed their knowledge of the meaning of integers and integer operations to construct and use tasks for teaching integers. All the sub-studies are qualitative in nature and serve to enhance our own understanding of the beliefs and practices preferred by teachers for the teaching of mathematics and the constraints on and the feasibilities of change in such beliefs and practices. In the concluding chapter, we provide a summary of findings from the four sub-studies and draw conclusions across them about teachers' practices, beliefs and knowledge

as well as the impact of professional development initiatives on participant teachers' beliefs and practices. Implications for professional development initiatives and limitations of the study are discussed. Recommendations for further studies have also been suggested.

2. Research background and context

2.1. Perspective adopted in the study: From 'training' to practice based professional development

In this study we adopt the view of the teacher as an "active learner" who is in-charge of her own learning through professional development opportunities that are meaningful. We acknowledge the knowledge and understanding that in-service teachers might have acquired through years of teaching, though they might not have thought critically about their own practices developed over the years. Many studies on teacher education have and continue to have a deficit view of teachers leaving them little opportunity in professional development contexts to participate as active learners and to connect what is being discussed to what they think and know (Roesken, 2011). We align ourselves with the view of professional development as promoting teacher's change as "growth" by recognizing the agency of the teacher (Day, 1999; Hannula, Liljedahl, Kaasila, & Rösken, 2007) and recognize the active role that teachers need to play in their own professional development for it to be meaningful.

In-service teacher education in India has for years been done through "teacher training". The training model is associated with the idea of "expanding individual repertoire of well defined and skillful classroom practice" (Little, 1993) while practice based professional development involves reflecting and discussion on the artifacts from the daily activity of teaching mathematics (Ball & Cohen, 1999; Matos, Powell, Sztajn, Ejersbø, Hovermill, & Matos, 2009). Building on situated learning theory, Matos et al. (2009) elaborate on how practice based professional development is promising as "the text of teaching serves as the context for teachers to learn about the specific aspects of their labor and reflection is expected to increase teachers' awareness of practice, allowing them to make thoughtful decisions in the immediacy of classroom work." We have used practice based pedagogy to engage teachers in their professional development in this study.

This study adopts the view of looking at in-service teachers as professionals who have been engaged in teaching for several years and thus possess situated knowledge of students

and the contexts in which they teach thereby viewing them as knowledgeable members in the community. However, we adopt a broader view of "community" as encompassing teacher educators and researchers along with teachers, all of whom are engaged in the enterprise of improving their practice by analyzing teaching and exploring alternatives to support students' learning (Jaworski, 2007; Jaworski, Goodchild, Eriksen, & Daland, 2011; Brodie, 2013). Even though teachers are members of the community of teachers, they may not have opportunities to communicate and discuss "teaching" in their everyday work, or to reflect and learn about teaching. Professional development of teachers in such a community encompasses articulating and sharing their beliefs and knowledge with other members of the community and thus participating in the process of knowledge construction by supporting and challenging the views articulated based on their experiences. Professional development is then a social process, that of development of a professional community comprising teachers, teacher educators and researchers rather than the development of an individual teacher who assimilates the materials and "methods" transmitted during a professional development program.

The interventions comprising this study embed the use of practice based artifacts (e.g. student work, blackboard work, textbooks, etc.) in professional development contexts like workshops, collaboration in classrooms and study groups. These professional development contexts serve as professional learning communities for teachers to engage with and reflect on their work of teaching mathematics. There are evidences in support of professional learning communities improving teaching practices and learning of students (Stoll, Bolam, McMahon, Wallace, & Thomas, 2006; Vescio, Ross, & Adams, 2008; Katz & Earl, 2010). Several research projects have made use of communities of practice in the form of professional communities, collaboratives (Nisbet, Warren, & Copper, 2003; Clark, Moss, Goering, Herter, Lamar, Leonard, Robbins, Russell, Templin & Wascha, 1996; Erickson, Minnes Brandes, Mitchell, & Mitchell, 2005), communities for inquiry (Jaworski, 2007) and even teacher networks in form of lesson study groups (Cerbin, & Kopp, 2006; Lewis, Perry, & Hurd, 2009), all of which have been found promising for providing an "ongoing venue for teacher learning" (Grossman, Wineburg, & Woolworth, 2001; p. 947). However, Brodie (2013) notes that "less research has been done on explicitly connecting the actual work of the professional learning community to shifts in teachers' practices". Some research studies which have looked at the issue have cited evidence for increase in

teachers' "confidence"(Graven, 2003; 2004) and attention to details of student thinking (Kazemi & Franke, 2004).

2.2. What aspects have been considered for professional development in this study?

Research studies have characterized teacher professional development as the development or change in teachers' beliefs (Kagan, 1992), knowledge (Ma, 1999; Ball & Cohen, 1999), goals (Schoenfeld, 1999), practices (Brodie, 2013; Kazemi and Hubbard, 2008) and in terms of development of teachers' identity (Graven, 2004; Jaworski, 2007) in the sense of "becoming" a mathematics teacher. In this study we study the interaction between beliefs, knowledge and practices of teachers as teachers participate in professional development opportunities in the three settings of professional development workshop, collaboration in classrooms and topic focused study group.

2.2.1 Teacher beliefs

It is important to study teachers' beliefs as they shape the way the teacher makes decisions during classroom instruction and thus impacts the understanding that students develop as a result of instruction (Wilson & Cooney, 2002; Schoenfeld, 1992). Also beliefs have been suggested as the clearest measure of teachers' professional growth (Kagan, 1992) and attending to beliefs is essential for improving teaching practices (Pajares, 1992).

Although beliefs have been considered as a "messy construct" (Pajares, 1992), there is general agreement in the mathematics education community that mathematical beliefs are "personal philosophies and conception about the nature of mathematics and its teaching and learning" (Thompson, 1992). For the purpose of distinction from beliefs, attitudes have been defined as "propensities towards certain pattern of behavior or emotional feeling" (Goldin, Rösken, & Törner, 2009). Bishop (1999) defines values to be behavioral aspects of "choosing" among alternatives, "preferring" and the "consistency of behavior". Beliefs have been associated with the conviction with which one considers a statement to be true.

The relationship between beliefs and practice has been studied but has yielded mixed results. Some studies have claimed that changes in beliefs are not reflected in practice (Fernandes & Vale, 1994). Several research studies report dissonances between teachers' beliefs about mathematics, its teaching and learning and actual classroom practices.

Various explanations have been offered for such inconsistency between “articulated beliefs” and “enacted beliefs” (Even & Ball, 2009) including that some beliefs are held more centrally than others (Pajares, 1992), or that the constraints and supports available in the teachers’ context allow teachers to enact some beliefs in consonance with their present purpose while assigning lower priority to others (Aguirre & Speer, 2000). Some have questioned the methodology for ascertaining beliefs which might consider teacher beliefs as inconsistent because of interpretations of some terms being different for the researcher and the teacher (Speer, 2005). This indicates that more research is needed to clarify the issue of relation between belief and practice (Ponte & Chapman, 2006).

Change or development of beliefs is recognized as a difficult and long term process (Clarke, 1994; Swan, 2006). Research also suggests that belief and practice are in dialectical relationship with each other and influence each other (Cobb, Wood, & Yackel, 1990; Guskey, 2000). Studies adopting ‘inquiry stance’ towards different aspects of practice like knowledge in curriculum, one’s idea about learning and teaching have been promising in terms of bringing about change and sustaining it (Farmer, Gerretson, & Lassak, 2003; Jaworski, 2008; Cochran-Smith & Lytle, 1999). Olson (1985) described how change in beliefs occurs by teachers reflecting and becoming aware of their practices and resolution of multiple interacting dilemmas and demands of their professional life. Several studies have found collaboration between the researcher and teacher in classrooms as having the potential to change teachers’ practice (Edwards & Hensien, 1999; Raymond & Leinenbach, 2000; Potari, Sakonidis, Chatzigoula, & Manaridis, 2010). However, it is important to not only understand what beliefs teachers hold but also *how* teachers hold these beliefs (Philipp, 2007). There is need to better understand the process of change and the challenges faced by the teacher to be able to support teachers in their professional growth that is meaningful for them.

2.2.2 Teachers’ knowledge

Teachers’ knowledge has been found to be directly correlated with students’ learning as it affects representations used by teachers as well as ways in which they deal with students’ errors. An important construct – pedagogical content knowledge (PCK) (Shulman, 1986) has been considered useful to explain effective teaching by understanding students’ thinking and difficulties while learning mathematics. The teacher’s role is thus important

in connecting children's mathematics to school mathematics. This is supported by Ball, Hill and Bass's (2005) analysis of mathematical knowledge required for teaching (MKT), which involves unpacking familiar mathematical ideas, choosing adequate explanations and representations, judging adequacy of alternative methods, and practice of problem solving. The knowledge of mathematical content for teaching can be distinguished from disciplinary knowledge of mathematics as being "mathematical knowledge unique to the work of teaching" (Ball, Thames, & Phelps 2008, p.4) and includes knowing "the source of a mathematical error" (Ball et al., 2008, p.7).

In this study, we have looked at various aspects of teachers' mathematical knowledge for teaching namely their knowledge of students' thinking, curriculum, specialized content knowledge of meanings and representations related to concepts being taught and pedagogical content knowledge. These aspects have been studied through analysis of teacher talk and the teachers' practices of questioning, explanation and evaluation. The attempt was to identify what knowledge gets articulated and used in different professional development settings and how it interacts with teachers' beliefs and practice.

2.2.3 Teachers' practices

There have been several studies which have studied change in teachers' practices through engagement in professional development using different lenses. Some have adopted the lens of mathematics that is done in classrooms, whereas some have used the lens of pedagogy, while some others have studied the pattern of interactions. Studies focusing on mathematics have looked for shifts from a focus on calculation to development of conceptual understanding (Thompson, Thompson, & Boyd, 1994) or "making meaning" of mathematics (Crespo, 2000, Empson & Jacobs, 2008). Studies which have analyzed questioning practices of teachers have distinguished funneling type questions from questions that involve students in thinking at higher levels (Sahin & Kulm, 2008).

Askew, Brown, Rhodes, William, and Johnson (1997) proposed three types of beliefs and practice orientations viz. connectionist, transmissionist and discovery. They posit that no teacher may exactly fit into an orientation and may combine characteristics of two or more orientations. In the study, we have analysed teacher's beliefs considering transmissionist and student centred teaching as the opposite ends of the continuum since connectionist and discovery notions were not identifiable in among the teacher participants. We have looked

at teachers' questioning, evaluation and explanation practices focusing on how these practices provide or constrain opportunities for students to engage with mathematics and the role that the teacher plays in that engagement.

3. Research study overview

The study reported here was located in a project aimed at promoting change in teachers' practice towards teaching that is more responsive to the development of students' understanding. The question investigated in the study is: In the context of the classroom teaching, what factors support teachers in adopting learner centered practices and what factors inhibit or constrain them in doing so. This question was interpreted in terms of a framework that took teachers' beliefs, knowledge and goals as the core components of teacher learning, giving rise to specific research questions addressed in four sub-studies, which are reported in the sections 4 to 7 of the synopsis. The methodological approach followed was participant observation and the methods of analysis were qualitative including case studies, supplemented with quantitative analysis for the belief questionnaire.

The participants of the study were mathematics teachers, who participated in professional development opportunities in three different settings: (i) orientation workshop for ten days using tasks situated in the work of teaching (ii) collaboration between the researcher and the teacher in the classroom with a view to support adoption of practices conducive for developing understanding of mathematics and (iii) Topic study group of teachers as an adapted form of lesson study where teachers explored meanings and representations of integers, made lesson plans, taught the topic in their respective classrooms and shared their learning in a workshop with each other and another group of teachers. The professional development activities during the two years of the study are depicted in the Table 1 below. The first two sub-studies were located in the workshop setting and focused on (1) teachers' beliefs and (2) design and interaction in the workshop during year 1. Sub-study (3) focused on one teacher's attempts to change her practice and was located in the classroom collaboration setting across the two years. Sub-study (4) in year 2 includes studying teachers' engagement with beliefs and knowledge in a topic study group through use of a framework for meanings and representations of integers.

Table 1: Timeline of the study

Sub-study	Year of the study	Data collected	Professional development activity
Sub-study 1	Year 1 (May-June, 2009)	Questionnaire and interviews	10 day professional development workshop
Sub-study 2	Year 1,(May-June, 2009)	Video of workshop sessions	10 day professional development workshop
Sub-study 3	Year 1 and 2 (2009-2010)	Audio records of lessons and researcher's notes of lessons and post-lesson discussions	Collaboration in the classroom
Sub-study 4	Year 2 (2010)	Video and audio records of workshop sessions; Audio record and researcher's notes of teaching of integers	Topic focused workshop + collaboration in the classroom

Following are the research questions addressed in each of the four sub-studies:

Sub-study 1: What are the preferred practices and beliefs held by the participating teachers and how do these interact with one another?

Sub-study 2: What principles of teacher professional development underlie the design and enactment of tasks during the PD workshop? What are the significant features of the participation (agency) by the teachers and teacher educators in the workshop?

Sub-study 3: What challenges does a responsive teacher face in implementing the intended change in practice?

Sub-study 4: How does teachers' participation in regular topic study group meetings support the development of their specialised content knowledge of meaning of integers and how does it influence their teaching in classrooms?

The three settings of the study are described briefly below:

Professional development workshop: 13 teachers from the government school system participated in a 10-day residential workshop along with 7 teachers from other schools (2009). The purpose of the workshop was to promote teacher learning through articulation and reflection on beliefs and knowledge. The workshop was designed based on the principles of situatedness, posing challenges to teachers' thinking and building a sense of community. More details about the workshop and an analysis of the workshop design and enactment is given in section 5. During the workshop, teachers' beliefs were assessed through questionnaire and interview, an analysis of which is presented in section 4.

Collaboration in the school: In order to provide support to the teachers in adopting practices conducive to developing understanding, and to understand the affordances and constraints in this process, collaboration with the teachers in the school setting was planned. In the thesis we discuss a case study of one such teacher.

Topic Study Group: The topic study group meetings were planned in the second year (2010) with the purpose of providing teachers with the opportunity to engage deeply with concept specific knowledge on a particular topic considered to be difficult for students, namely, integers. We tried to engage teachers in learning from the analysis of textbooks and through engagement in tasks for understanding the meanings that can be connected with students' experience. Teachers also constructed and explored alternative tasks and approaches for teaching the topic.

3.1. Participants

Participants in the study were mathematics teachers teaching primary and middle grades in a nation-wide Government school system and were nominated by their principals to participate in the study. All the teachers who participated had more than 15 years experience of teaching and were between the age range of 39 to 50 years. Of the 8 middle school teachers 3 were males and there was no male primary teacher.[1] During the course of the study, classrooms of 4 teachers (1 Primary and 3 middle school teachers) were visited. Analysis of one primary teacher who was visited across the two year study is reported as a case study in the thesis in Chapter 6. Reports of teaching integers of the other three teachers have been analyzed and presented in chapter 7. Of the 13 teachers from the system who participated in the professional development workshop (2009), 4 primary and

1 The number of male teachers at the primary level is much smaller than female teachers in this school system.

4 middle school teachers were local i.e. from the same city and participated further in the study. All the 8 local teachers participated in the 6 one day topic study workshops in the second year (2010) after the first workshop. The analysis of the group of four middle school teachers for topic study group on integers has been reported here. The primary teachers group was not able to implement their plans properly in classrooms due to constraints of the timetable and upcoming examinations.

4. Beliefs and practice

The sub-study described in Chapter 4 of the thesis analyses the questionnaire responses of 26 teachers and interview responses of 11 teachers from the same school system, who participated in the orientation workshop. We present an analysis of the data collected through a questionnaire designed to assess beliefs of teachers regarding mathematics, teaching of mathematics, students and self along with the practices they preferred for teaching mathematics. Additionally, data from teacher interviews are compared with this data and the differences and similarities in teachers' responses in the questionnaire and the interview are described. The open coding of interview responses also threw light on how teachers have interpreted the questionnaire statements. Together, data from questionnaire and interviews have been used to make claims about the beliefs teachers hold and the practices they prefer for teaching mathematics. Although data from classroom observations have not been used, teachers' description of preferred practices reveal the images teachers have about their own teaching and their alignment or non-alignment with the articulated beliefs.

4.1. Framework for analyzing beliefs and practice

The framework that we have used to analyze teachers' practice is the continuum across the dimension of transmission based teaching and student centered teaching where the student-centered end views the teacher in an active role basing their pedagogical decisions on students' thinking. We introduce a distinction between core and peripheral practices of the teachers, which are in turn related to core and peripheral beliefs held by teachers. Core beliefs are expressed strongly in teacher responses or inferred by considering the tensions between the beliefs and practices reported by the teachers. Questionnaire responses indicated alignment towards a student centered view but the interview responses revealed closer alignment to the transmissionist view. This is because teachers elaborated on

examples and gave explanations of the terms used during the interviews, while their interpretations of questionnaire statements remained hidden. Teachers incorporated some student centered practices into their repertoire as peripheral practices, while core practices were transmission based. The interviews also revealed the gaps in teachers' thinking about the purpose of student centered practices and showed that they had limited knowledge of why and how procedures work.

4.2. Core and peripheral practices

Analysis of data across questionnaire and interview, showed four core practices that were preferred among the teachers in the group and were reportedly used regularly by most teachers. These were teaching by showing procedures or solved examples, giving students repeated practice of solving problems, focus on speedy solutions through teaching shortcuts and close following of the textbook by doing exercises and problems. Peripheral practices were used less regularly and were given less priority by teachers. They included use of activities for introducing a topic or to help remember a procedure, focusing on explanation and justification, and connecting students' everyday experience with mathematics done in classroom.

4.3. Core and peripheral beliefs

The findings about practices indicated that the core beliefs held by teachers about mathematics, teaching and students, showed greater consistency with the transmission view of teaching and procedural view of mathematics. Teachers' insistence on practice for learning mathematics also pointed towards the belief that learning mathematics calls for memorization. Most believed that it is not possible for students to come up with mathematical ideas on their own without being taught (Kumar & Subramaniam, 2013).

Teachers recognized the role of justification and reasoning in school maths but still considered maths as restricted to learning the four operations. Tensions were evident in teachers' talk about focus on procedures versus focus on reasoning. Teachers' interviews indicated limited knowledge of why procedures work and their justification, as well as resistance to alternative procedures different from the standard algorithms. Teachers considered mathematics as difficult for students and tried to make it easy using concrete materials and activities to make it interesting. However topics in higher grades like algebra and geometry were considered as difficult to represent through concrete material. Teachers

showed a positive attitude towards using contexts from daily life but used them as descriptions of the problem and rarely focused on the mathematical meanings within contexts. Teachers gave more socially appropriate responses to questionnaire items on class and gender bias. However, in the interview teachers talked about how they have lower expectations from poor or girl students and that they focus on repeated practice and memorization of problems likely to appear in the exams to make weak students pass. Teachers who had positive experience with maths in their school education were critical of the lack of practice exercises in the new textbooks while some of the teachers, especially primary teachers who had had unpleasant experiences were critical of the widely prevalent practice of rote memorization in learning mathematics. Teachers talked about the pressure to get 100% pass percentage results in examinations.

4.4. Interaction between practice and beliefs

We found that core beliefs together form a coherent stable structure, as these beliefs are in alignment with each other and support the adoption of related practices. For e.g., the core belief of viewing mathematics as consisting of procedures and learning as memorization of these procedures is reinforced the practice of teaching procedures and repeated practice for memorisation. Further, such practices and beliefs are at the core of the teachers' identity as they construct their sense of self from their students' performance on the tests and exams which evaluate their capacity to remember the procedure to solve a particular problem. Teachers' years of experience of learning and teaching mathematics focused on procedures supports the transmissionist view further and adds stability to this core belief structure. It makes the belief structure resistant to educational reform efforts where change is sought through the change in textbooks and issuing of circulars by authorities. Strong beliefs about procedures and memorization constrain the change while belief about maths as abstract made teachers integrate a few practices only superficially.

The reported practices are cognitive images of how teachers view their practice rather than objective descriptions of their practice. Therefore these are indicative of beliefs held by teachers since it involves some generalization and reflection by the teacher to report their teaching. Core beliefs are reflected in the core practices, while articulated beliefs which are not reflected in practice or were not given due importance by teachers might be more peripheral in nature. We found that much of the inconsistency, conflict and tension between

beliefs can be inferred even from reports of practice, and not only from observations of actual practice.

5. Principles of design for the workshop

In-service teacher development in India has been driven largely through frequent orientation workshops held for teachers to make them familiar with the expected curriculum and pedagogy as visualized in the curriculum reform documents. Although, there has been criticism of such a mode of teacher development, workshops continue to be primary intervention site for TPD. Hence, it is important to analyze the design and enactment aspects of a workshop in the light of its goals to identify opportunities provided for teachers' reflection and learning (Kumar, Subramaniam & Naik, 2013). In sub-study 2 reported in chapter 5 of the thesis, we analyze the data from a ten day professional development workshop for the study participants. The goal of the workshop was to strengthen teachers' professional knowledge and to provide opportunities to reflect on their underlying beliefs. The analysis of the workshop was undertaken to identify elements that were aligned with this goal (Miles & Huberman, 1994).

5.1. Framework for design and enactment of the workshop

In this chapter, we articulate a framework consisting of principles informing the design and enactment of a professional development workshop that addresses teachers' knowledge and beliefs. The principles are retrospectively reconstructed through the analysis of the workshop tasks and enactment. We describe the types of sessions conducted and select for analysis episodes which presented opportunities for teacher learning. The analysis of workshop sessions was done through open coding of the session transcripts followed by development and refinement of the codes to identify features of the task, evidences of teachers' agency and teacher educators' agency in workshop interactions. The analysis led to reconstructing the underlying principles of task design and enactments that contributed to the emergence of opportunities for teacher learning. These principles were not explicitly articulated as design principles prior to the workshop although they were largely implicit in its design and implementation. The principles of design of the workshop that were identified were (i) situatedness in the work of teaching, (ii) offering challenges to teachers to revisit their knowledge and beliefs, and (iii) developing a sense of belonging to a professional community. Subsequently, analysis of workshop sessions is presented with

examples of episodes to illustrate – (a) how the three principles and goals of the workshop design shaped the tasks and enactment of those tasks, (b) how situatedness of the tasks and specific aspects of enactment led to exercise of agency by teachers and teacher educators and (c) how teachers exercised agency by contributing their own professional knowledge and by challenging and reflecting on their beliefs and practices . We describe the efforts made to situate the discussion of teaching and learning both in the context of the work of classroom teaching and within the community of teachers.

5.2. Principles embedded in task design and enactment for the workshop

The tasks used in the sessions drew on artifacts and teaching contexts and were focused on mathematical or pedagogical issues that arise in the context of teaching or have implications for teaching mathematics. As teachers were able to relate the artifacts to their practice, they shared their knowledge and beliefs about teaching mathematics in engaging with the tasks. The challenges embedded in the tasks like analyzing student work to explain why a student responded in a particular manner, identifying what a student knew and did not know, led to teachers revisiting the taken-as-granted practices in every day teaching like repeating the procedure again in case of error. The discussions on interesting examples of student thinking and the complexity of teaching helped in establishing the fact that developing knowledge of teaching is one of the main objectives of the community of which teachers, teacher educators as well as researchers are an important part.

5.3. Teachers' agency in workshop interactions

The engagement of the teachers in the sessions involved not only sharing the practices adapted by them in their teaching but also bringing to bear their own professional knowledge through conjectures, assertions, counter-arguments, etc. Further, the discussion frequently led to articulation and reflection on beliefs held by the teachers. They were able to reflect on their own teaching as one of the sources for creating misconceptions among the students. However, their identification of conceptual gaps in the students' thinking was constrained by their own limited knowledge of concepts.

The analysis of the types of teacher engagement that occurred during the episodes throw light on the kind of opportunities that arose for teacher learning. Teachers' engagement took the form of anticipating and predicting students' responses, identifying key knowledge

pieces, conjecturing underlying causes, articulating and contesting beliefs and assessing a teaching resource or a teaching approach (Kumar & Subramaniam 2012a). Such engagement was crucial in building shared understanding not only among teachers, who rarely get opportunities to reflect collectively about teaching in their schools, but also for teacher educators by providing windows into teacher thinking. Teachers' assertions, counterarguments, alternative explanations and assessments were also a resource, which deepened fellow teachers' and teacher educators' understanding about mathematics teaching as it takes place in classrooms. Teachers' learning from the workshop was reflected in the written feedback that they gave about the workshop as well as the revisions that the teachers made in their responses to the questionnaire when they attempted the questionnaire again at the end of the workshop.

5.4. Teacher educators' agency in workshop interactions

The teacher educator's agency was reflected in the way the teacher educator engaged teachers in discussion. Looking at two contrasting episodes by two different teacher educators, we found that re-voicing and making arguments using teachers' assertions indicated high inter animation in the session and showed increased participation by the teachers. Another factor that supported high inter-animation was the manner in which questions were posed and responses were evaluated by the teacher educator, which shifted the authority of evaluating the opinions to teachers. The teachers educators' beliefs and goals for the workshop as well as knowledge of the teacher's context were visible in the framing of the questions and moves to engage teachers in the discussion. Where necessary, the teacher educator was able to provide alternative viewpoints to initiate cognitive conflict to make teachers think and reconsider alternative views in their thinking. The teacher educator's efforts to connect the specific discussion in the sessions with the broad goals of teaching of mathematics or about mathematics education were important. Thus, an analysis of the teacher educator's moves show alignment with the goals of the workshop and led to teachers' participation in sharing their knowledge and revisiting their beliefs.

5.5. Implications of the framework of Principles for design and enactment of workshops

We have attempted to present a framework that can be applied to an analysis of the components of a TPD workshop and interaction episodes and can identify design as well as

enactment aspects. The framework does not describe what constitutes knowledge for teaching mathematics, nor does it elaborate on the nature of beliefs conducive to teaching for understanding. A framework that elaborates on the specifics of knowledge and beliefs relevant to teaching mathematics will need to be contextualized with regard to topics and to teacher communities. The framework presented here, in contrast, identifies certain principles that are important for the design of tasks and their enactment in workshop sessions. We believe that this framework would be useful in identifying and providing rich descriptions of elements that are important in a TPD intervention (Kumar, Subramaniam & Naik, 2015a).

6. Role of beliefs and knowledge in teaching of fractions: Case study of Nupur[1]

Nupur was a mathematics teacher teaching the primary grades IV and V in a government school at the time of the study. She was a participant in the ten day orientation workshop of the study and had frequently expressed an intention to change to a more student-centered teaching. The case study of Nupur was considered as potentially useful since it could provide insights about the extent to which holding positive beliefs for student centered teaching can motivate a teacher to explore new practices and the kinds of challenges such a teacher might face. Nupur's responses to interview questions and the questionnaire indicated that she believed that teaching should be focused on mathematical concepts and that mathematics done in the classroom should be connected to the students' everyday experience. This chapter discusses the analysis of lessons taught by Nupur on the topic of fractions post the professional development workshop. The researcher participated in her teaching as an observer and a collaborator.

6.1. Framework for analysis of teaching

To analyze the teaching of fractions we have used the framework of fraction sub-constructs (Kieren, 1976; 1988) since it illuminates about the different meanings and contexts that help to develop a wholistic understanding of fractions. Kieren (1988, 1993) identified five sub-constructs of fractions namely part-whole, share, measure, operator and ratio which can be illustrated in different contexts and representations. He argued that children develop an impoverished concept of fractions as a result of being exposed to only those contexts

1 Name changed

and representations which exclusively use the part-whole meaning of fractions. We have analyzed how and which sub-constructs of fractions were used in the task framing and task implementation that occurred in selected lessons focusing on equivalent fractions across two years of the study. There were various instances when teacher did engage students in a conceptual discussion. The analysis revealed that the teacher's specialized knowledge of fraction representations and meanings shaped the way the teacher selected the tasks and managed the interactions during the discussion of the task.

The sub-construct theory of fractions was also used to analyze the textbook chapter. We found that most of the tasks in the chapter correspond to the part-whole meaning of fractions using area representation. The measurement meaning was used to a limited extent.

6.2. Task framing

The analysis of the lessons taught by Nupur on fractions indicated a gradual shift from using only the part-whole meaning of a fraction to also using tasks involving the measure and operator interpretation of fractions. The initial tasks were based on area representations with or without use of contexts to talk about what fraction represented a part given. Usually during a lesson, Nupur would start by taking up the textbook task and discussing its context but followed it up with a similar calculation task. She rarely used a context on her own to frame the problems. Gradually, the basis of selection of tasks shifted from what might be interesting for students to do to what is necessary for building students' understanding. In later lessons she constructed impromptu tasks based on how students responded in the class. In some lessons, she gave students choice in several tasks and asked students to construct questions on their own after solving a few questions. However, she was not able to fully anticipate or address the challenges that students faced when they constructed tasks on their own. Nupur also started using open ended tasks in later lessons and allowing students to come up with their own ways of solving problems. The challenge was, however, to be able to respond to students appropriately when they shared incorrect strategies.

6.3. Task Implementation

Analysis of task implementation across the lessons indicated how the teacher's practices related to questioning, evaluation and explanation underwent a gradual change as a result

of teacher trying out new practices while collaborating with the researcher for teaching fractions in the classroom. Such changes were not stable as Nupur reverted to older practices when faced with challenges.

In terms of questioning practices, the change occurred in the nature of the questions that were asked, the way the representation was used to pose the task or discuss the solution and the variety of contexts used to pose the questions. In the initial lessons Nupur asked many funnel type questions which had only one correct answer. She encouraged students to use the double counting method to name the fraction by counting the number of shaded parts and the total parts. She considered students' misconceptions and errors as careless mistakes and tried reminding them of the method to arrive at the correct fraction by double counting.

In later lessons, the teacher became conscious about making students see that equal parts of an area can be made in different ways and contiguous or noncontiguous parts can still denote the same composite fraction. She focused her attention on addressing students misconception of reversing numerator and denominator while naming fractions, comparing fractions and comparing fractions with whole numbers.

The representations used in the tasks and questions posed by the teacher and the way the teacher used the representation also changed. Initially, she focused student's attention on how the representation of 1/2 looked to make judgements about equivalent fractions. Later she tried to focus students' attention on finding the size of each part by iterating the part to measure out the whole.

While questioning Nupur also tried establishing equity in the classroom by asking equal if not more questions to girls and giving them time to think by asking boys to be quiet for some time.

Nupur's practice of evaluation also exhibited a change from the teacher being the authority to determine the correctness of the answer to asking students to vote for the answer that they thought was correct from among the different answers shared in the class. She moved on from considering the correct answer as indication of understanding to asking students to explain their answers. However, students' explanations were procedural in nature which according to Nupur, they had learnt from attending tuition classes.

From the above discussion, it is clear that her practices related to the kind of explanations

discussed in the classroom underwent a change. She tried to develop explanations based on visual representations, focusing on concepts and engaging students in giving explanations and reasons for using certain procedures. However, there were times when she struggled with developing accessible explanations and establishing connections between the actions carried out on a visual representation with the procedures carried out on a symbolic representation. For e.g., she was not able to connect finding 2/5 of 100 by selecting 2 out of every 5 circles drawn from 100 circles and multiplying 2/5 with 100. She also quickly shifted from using area representations of ½ to finding equivalent fractions of half by doubling both the denominator and the numerator without discussing why it would yield equivalent fractions. She initially focused on perceptual features of visual representations by asking students to draw representations which "looked like ½". However, she shifted to asking students to measure the whole using a part denoting the unit fraction. In all these efforts at changing questioning, evaluation and explanation based practices, the researcher played the role of a collaborator. The discussions between teacher and the researcher usually focused on students' thinking, planning for teaching and modeling certain practices within the classroom (Kumar & Subramaniam, 2012b).

6.4. Interaction between beliefs, knowledge and practice

The examples given in this chapter, illustrate how conducive beliefs towards student centered practices set the stage for the teacher to appropriate strategies like establishing connections with daily life, asking 'why' questions, discussing different answers of students, building explanations based on students' responses, using representations and concrete materials to develop reasoning and pursuing students' responses and conjectures further to develop understanding of mathematics. This indicates that when a teacher starts to incorporate student centered practices because of some conducive beliefs, support is needed to sustain the change in practices. The most important support identified is the development of teachers' knowledge regarding knowing how to act in situations which may be unanticipated as well as knowing the content in flexible ways to link with students' understanding. This knowledge helps the teacher to pursue the classroom discussion beyond just establishing the answer as right or wrong, pursuing students' conjectures and responses to develop important mathematical concepts allowing her to build explanations based on students' responses and above all basing the teaching on students' thinking rather than what is given in the textbook.

For teaching to be determined by the student's thinking, a teacher has to be empowered to take decisions like selecting appropriate examples for teaching, recognizing opportunities from student's responses to develop important concepts, constructing questions for assessment of understanding and also determining which responses of students should and should not be considered as indications of understanding.

7. Topic focused professional development: The case of integers

Sub–study 4, reported in chapter 7 of the thesis, identifies and analyses the topic specific specialized knowledge of a group of 4 middle school in-service mathematics teachers (Rajni, Swati, Anita and Ajay[2]) on the topic of integers . It tracks the growth of such knowledge as a result of participation in "Topic focused workshops" and its impact on teaching. The 6 one-day workshops were held over a period of five months, at the end of which the four teachers presented their learnings to a group of teacher colleagues. The results show how participation in these workshops helped the teachers in developing specialized content knowledge in the form of meanings, contexts and representations related to integers and integer addition and subtraction, through development of tasks and resources for teaching integers. The teachers also developed deeper criteria for selecting representations based on meaningfulness and consistency of meanings across representations and grades. Teaching of the topic of integers overlapped with the workshops and the teachers were motivated to use the resources developed in the workshops in the classroom. Teachers exhibited a shift in teaching from exclusive reliance on rules to a focus on improving reasoning and understanding, and increasing the diversity of students' participation in the classroom.

These findings are based on an analysis of teachers' talk in the workshops, the reflections that they shared with teacher colleagues, the lesson plans made by teachers for teaching integers and analysis of teaching done by three of the four teachers. The data was collected in the form of video and audio recordings of workshop sessions and teachers' classroom teaching (Rajni, Anita and Swati). These recordings were transcribed by the researcher. A few lessons from classroom teaching of Rajni and Anita could not be transcribed and researcher's notes were used along with reviews of audio recordings to analyze the

2 All names changed

teaching. The transcripts were analyzed at first through open coding method and writing of memos. The codes were refined to identify and analyze teacher talk in the workshops and classroom, where teachers used different meanings and representations in relation to integers.

7.1. Integer meanings and representational adequacy

A framework of specialized content knowledge (SCK) (Ball, Thames, & Phelps, 2008) for the topic of integers was used to design resources in the workshop. The framework, which was developed on the basis of a review of research studies on the learning of integers is presented in Table 2. The framework takes into consideration the meanings of the negative sign used in symbolic expressions, the meanings attributed to integers in contexts as well as the models used for ordering integers and operations on integers. The negative sign in symbolic expressions can denote unary (–7), binary (2 – (+7)) and symmetric function (–x) (Vlassis, 2004; 2008). In contexts of application, integers can represent state, change or relation between quantities (Vergnaud, 1982; Kumar, Subramaniam, & Naik, 2015b). Models like the number line and the neutralization model are used by teachers to help make sense of the comparison of integers and operation on integers (Stephan & Akyuz, 2012). We have used this framework to analyze teachers' discourse concerning teaching of integers in the workshop and in the classroom.

Table 2: SCK framework for integers

Meaning of the negative sign	Meaning of integers in contexts	Meaning of addition subtraction of integers
Unary function, Binary function, Symmetric function	State, Change, Relation	Combine, Change, Compare
Models: Number line models / neutralization models		
Contexts: Eliciting salient quantities and derived quantities		

Using the framework discussed above, analysis of the textbook chapter on integers for Grade 6 (which was the focus of the study), revealed that the symbolic representation and the number line representation were predominant in the textbook. Further, integers were used to represent 'state' in most of the tasks in the textbook, and the integer meanings of 'change' and 'relation' did not emerge adequately.

In addition to the above framework on integer meaning, we have used a framework to analyze the criteria evident in teachers' talk for selecting and using representations for teaching integers. We have identified three dimensions of what teachers consider as "representational adequacy" (diSessa, 2002), namely, translatability, consistency-coherence and meaningfulness of representations. We found that the criteria teachers applied shifted from surface level concerns to deeper concerns for representational adequacy by applying all three criteria in a coordinated manner.

7.2. Extending the range of integer meanings and contexts

When teachers were asked initially to think of contexts for representing integers, they proposed several examples which did not really need integers and could be represented using whole numbers or were restricted to the 'state' meaning for integers. In subsequent workshops, the teachers were exposed to the integer meanings of 'state', 'change' and 'relation' through worksheet tasks. A large number of contexts were discussed, integer meanings explored and a judgment was made about their pedagogical usefulness. Teachers engaged in discussions of contexts that use change and relation meaning of integers like positive and negative scores in a quiz competition and change in baby's weight. They were able to distinguish the two meanings of minus sign: indicating subtraction and as a sign for negative integers. Teachers were able to identify and give examples of relevant contexts which involve use of different meanings of the minus sign – subtraction of whole numbers, representing negative integers and subtraction of negative numbers. They also recognized that these distinctions posed a challenge for students to develop an understanding of operations with integers. The exploration of meanings in relation to contexts using the integer meanings framework led to teachers describing an increased variety of situations that can be represented by integers. This contributes to increase in the richness of the example space (Watson and Mason, 2005) that teachers can access for generating tasks, guiding classroom interactions and assessing learners' understanding. Not only more

contexts but context features other than ‘state’ were also represented by integers within the same context. Teachers thus began to move from initial beliefs that symbolic representations are more efficient than other more concrete representations like models or contexts. Initially teachers used contexts only to introduce integers and the need for integers, but not for integer operations. In the workshops, teachers began to explore the use of contexts to explore and learn integer operations.

In analyzing a variety of contexts, the teachers used integers to represent *derived quantities* that were different from the salient quantities – for example, change in temperature as opposed to temperature, change in baby’s weight as opposed to weight, and relative position in the mall as opposed to floor number. This led teachers to design and adopt such contexts where integers represent change and hence could be added meaningfully – hourly change in temperature, weekly change in a baby’s weight and movement of a lift in an “integer mall”. Interpreting an integer as representing a “static relation” allowed further exploration of contexts and the possibility of modeling the subtraction operation using contexts. In the integer mall, for example, subtraction was used to find the movement required to reach a target floor from a given floor. The teachers’ shift from using integers to represent only states to representing transformation and relation is an important move, whose significance and challenge has been identified by other researchers (Thompson & Dreyfus, 1988).

Merely becoming aware of the various meanings of the minus sign, of integers and of integer addition and subtraction does not constitute SCK for teaching mathematics. Using the framework of meanings, teachers need to construct further elements of SCK by relating it on the one hand to teaching concerns and on the other to representations. We found evidence of three ways in which such construction of SCK was made by teachers in the workshops. Firstly, as we have mentioned before, teachers identified features and processes associated with representations, especially contexts, that corresponded to one or the other meaning of integers. Secondly, teachers connected various meanings of integers through their insight about the key idea of a reference point. They noted that in contexts where a sequence of changes is represented by integers, the reference point is constantly shifting. They noted that to represent state using integers, they need to fix a “zero” as a reference point by convention, while to represent relations, the reference point is arbitrary. The teachers also made connections across different layers of meaning, by relating the

distinction between the two meanings of the minus sign (integer and subtraction operation) to the distinction between the state and change meanings of integers. Finally, teachers used the framework of meanings to interpret student errors (the difficulty in extending the take away meaning of subtraction to "taking away" a negative integer), to offer explanations using representations (moving right on the number line corresponds to an increase) and finding new ways of modeling procedures for addition and subtraction using representations (subtraction using the neutralization model, or using the "integer mall") (Kumar, Subramaniam & Naik, 2015b).

7.3. Refining the criteria for representational adequacy

Analysis of the teachers' talk in the workshop indicated a shift in the criteria applied to evaluate the adequacy of representation used to teach integers. Initially, the teachers preferred the symbolic mode and the use of representations by teachers was limited to those given in the textbooks thus exhibiting limited specialized content knowledge (SCK). Teachers' criteria for evaluating representations were based solely on translatability but did not show much concern for meaningfulness or consistency of meaning. The teachers' discourse shifted from attributing students' errors to students' failure to memorize, recognizing that instruction too can lead to errors, and that the meanings that students associate with symbols change as they learn new topics like integers. This made teachers value the consistency in meaning across different representations used for purposes of explanation. The discourse around representations thus deepened to establish connection with meanings and even leading to revisions in teachers' explanations.

Teachers were able to make connections between different representations like number line and an "integer mall" with a lift, using the framework of integer meanings. They were able to interpret movement on the number line as increase or decrease, and to provide a more meaningful explanation of why additive inverses sum to zero (Bajaj & Kumar, 2012). Thus meanings helped in bringing coherence among different representations that could be used for teaching integers as well as an increase in a variety of contexts for teaching. The discourse of teachers after engagement with the framework of meanings showed deeper concerns for translatability, meaningfulness and consistency of representation and their use.

The shift that teachers exhibited to a deeper level of concern for representational adequacy

from a surface level could be due to the type of interactions in a collaborative setting. Teachers' explanations of using representations were critiqued, challenged and subjected to analysis as to how they represent different facets of integer concept as well as discussions about why the particular representation works. Teachers were also exposed to a variety of representations and their explanation and justification given by their colleagues and the teacher educators. This pushed the need to consider their choice of representation more deeply, and the capability to critique and design representations. So the collaborative discussions helped build both knowledge of representations and knowledge about representations.

7.4. Impact of topic focused professional development

Teachers' learning from participation in these workshops was visible across two dimensions. One was the way in which there was a shift in teachers' goals and beliefs and the other was the extent to which teachers' went beyond the textbook and used the problems designed during the workshop. In their lesson plans and report of teaching experiences, three teachers (Swati, Anita and Rajni) acknowledged a change in their approach from telling rules in the beginning to exploring contexts first with students and then inducing rules. Another shift in the goals was from avoiding student mistakes to understanding the thinking behind student errors (Kumar & Subramaniam, 2012c). The shift in beliefs about teaching integers through rules versus engaging students in reasoning involved teachers experiencing the struggle of managing classroom interactions to develop understanding while developing their own knowledge in order to support students' engagement in reasoning. Teachers' reflections indicated that they went beyond the textbook by using tasks constructed in the workshop to engage students in thinking and reasoning rather than having them solve the tasks mechanically using a known procedure (Kumar & Subramaniam, 2015).

From the observation of teaching, it was evident that teachers did use the contexts and key ideas discussed in the workshops, although in different ways and with different degrees of integration with the textbook exercises and teaching of rules. Among the range of representations discussed in the workshop, teachers used the vertical (integer mall context) number line together with the integer mall context to discuss the meaning of integers, ordering and comparison as well as for addition and subtraction of integers. They found

subtraction most challenging to discuss using the number line. They also used the neutralization model to discuss addition of integers. Anita, went a step ahead to use the context of team scores in a quiz competition to discuss addition in which teams got a positive score for correct and negative score for wrong answers.

All the teachers used the meanings discussed in the workshop during their teaching. Since the textbook had mostly examples and tasks which represented states as integers, this meaning was used by all teachers to a greater extent than others in all the representations and contexts. All teachers used the meaning of change to discuss addition of integers in both number line and neutralization model. However, Anita used the meaning of opposite, change and relation with zero to argue for –1 being placed below zero in a vertical number line. Using the meaning of integers as relation, zero was discussed as a placeholder on the number line to indicate the equal and opposite relation between a positive integer and its opposite negative integer. Zero also acquired the meaning of no change when the addition of positive and negative integers cancelled each other. The idea of zero being greater than an infinite set of negative integers was also discussed by Anita.

Although teachers used different representations and meanings in their teaching, there was back and forth movement between discussing meanings using representations and contexts, and using rules to find answer. There was also pressure from students to use rules since they were already familiar with them from their tuition (coaching) classes. The main reason that teachers cited for not being able to implement the lesson as they had planned was that they faced the pressure of syllabus completion and resistance on part of students to engage in meaning based discussion using contexts. Teachers too find it challenging to engage and respond to students in discussions about meanings of integers and to reason using representations and contexts. However, they identified students' responses which indicated the need to understand meanings and shared them in the workshop discussion and presented them to other teachers.

Given the slow nature of change in developing knowledge to use representations meaningfully and to use practices to support meaning making in classroom, it is understandable that teachers showed back and forth movement using ideas developed in the workshops and the ideas that they had been using for teaching integers. We note that all the teachers in the study were highly experienced, knowledgeable and resourceful and had many years of teaching experience. They were aware of student errors and were familiar

with the textbook and the curriculum. Given this background, the lack of detailed attention paid by the teachers to issues of meaning in the initial phase of the workshop was remarkable. It suggests to us that the knowledge encoded by the framework of integer meanings is an important part of SCK that is not gained directly through the practice of teaching alone. One reason for this might be that developing such distinctions and frameworks needs deep engagement with issues connected with both content and with the learning of content. Hence SCK elements such as integer meanings may be important bridges between the knowledge acquired through mathematics education research and the knowledge that is essential for effective teaching.

8. Implications

8.1. Implications for professional development design

The sub-studies indicate the features in the professional development design that worked in promoting professional growth of teachers as well as aspects that can constrain teachers from adopting practices that support understanding mathematics. In-service professional development has suffered in India due to fragmented efforts which are neither perceived to be useful in the classroom, nor have been able to address teachers' concern for developing students' understanding of key and foundational ideas in mathematics. The sub-studies have several implications for designing of professional development opportunities which are listed below.

1. In all the sub-studies there are indications of interactions between the beliefs held by the teachers and the practices preferred. Sub-study 1 showed how without change in core beliefs, the practices advocated by reform documents get incorporated as peripheral practices. However, beliefs about focus on reasoning and use of familiar contexts can serve as stepping stones towards adoption of practices that support understanding mathematics as indicated in sub-study 3 and 4. These practices may become central to teachers' practice when teachers develop appropriate knowledge to support use of these practices. Teachers will thus need opportunities to reflect on the beliefs held by them and the development of specialized knowledge for teaching mathematics in both professional development contexts and in classrooms.

2. Studies also indicate that teachers' knowledge of content, meanings, students' thinking, representations influences the interaction between beliefs and practice. In sub- study 1 when teachers exhibited limited knowledge of representations and why algorithms work, they had beliefs more aligned towards mathematics as procedures and teaching as a transmission based activity. However sub-study 2, 3 and 4 show instances of teachers expressing and developing their knowledge of meanings and representations through engagement in tasks in workshop and through efforts to focus on reasoning and development of meanings in their teaching.

3. It is important for teacher educators designing professional development interventions for teachers to anticipate the teachers' common beliefs and practices and to create opportunities for teachers to reflect on them. The framework developed in sub-study 2 of situatedness, challenge and community building can be used to design the sessions and also manage the interactions within the workshop. This can be supported through use of artifacts from teaching, making teachers articulate their beliefs and knowledge and asking teachers to explain student thinking as done in sub-study 2. Another possibility is to encourage teachers' make the criteria for evaluating the resources of teaching explicit and these shared criteria can be established through negotiation, as was done in sub-study 4.

4. Even when a teacher is open to bringing about change in practice and is reflective, he/she might face challenges in actually changing teaching practice. These challenges can vary from identifying the gaps in the teaching and use of representations, to the knowledge of meanings, contexts or representations, to negotiations of the norms of the classrooms to making students participate actively as reflected in sub-study 3 and 4.

5. The theoretical framework used in sub-study 4 was derived from the research literature. The sub-studies have illustrated the knowledge that the participating in-service teachers have about the common mistakes and knowledge of key ideas and explanations which have been discussed in research papers. Therefore, the sub-studies would hopefully contribute towards building a balanced vision of status of teachers' knowledge which need to be elicited and built upon in professional

development space rather than glossing over the deficits.

6. The teachers' learning in sub-study 4 indicates that topic focused professional development is needed to develop in-depth knowledge through activities like textbook analysis, analysis of students' errors, analysis of meanings, representations and explanations, and collaborative lesson planning and sharing. While this kind of exercise cannot be expected from teachers for all the lessons, focusing on one topic each year in this integrated manner will help in developing teachers' specialized content knowledge in integration with other knowledge, while also contributing to the knowledge of teaching within the community of mathematics educators.

7. The two topics of mathematics that have been focused in this thesis are fractions and integers. There are implications for designing PD for these topics for mathematics teachers. Firstly, teachers need to be aware of the different meanings that these concepts may have in different contexts. Secondly, teachers need to make the connections between these meanings and the representations used in teaching and the tasks used in order to support meaningful discussion using them. Thirdly, the understanding of meanings and representations need to be connected with the standard symbolic procedures, so that teachers are able to unpack it for the students and engage those students who know and perform procedures mechanically without understanding them. Fourthly, the teachers also need to understand why rules work and how to make shifts from teaching using contexts and representations to developing an understanding of rules and other generalizations that can be made about mathematical concepts. Lastly, teachers need to integrate specialized content knowledge with the knowledge of students, applying both to designing tasks to develop understanding.

8.2. Limitations of the study

This research study being exploratory in nature is bound to have limitations, some of which are listed below.

1. The sample selected for the study is small and primarily from Mumbai city for sub-study 1 and 2. For sub-study 3 and 4 smaller samples were selected from the initial sample as the nature of the research question required getting an in-depth understanding of the

challenges faced by teachers while teaching and the intervention in the form of a topic focused workshop. The qualitative nature of sub-studies 3 and 4 justifies the small sample. The findings of the studies are thus indicative and not generalizable across the population of teachers although they involve rich descriptions of teachers' struggles in adopting new practices in classroom.

2. A large part of the data in this study is in form of audio recordings which required transcribing. To limit the scope of the study and time required to transcribe and analyze, there were several lessons which had to be dropped from the analysis. For sub-study 3, only lessons related to fractions were considered since it was one of the topics focused during sub-study 2 and one could analyze teaching across two years. For sub-study 4, lessons related to integers were selected since they informed the impact that topic study workshops had on teachers' practice.

3. It was not possible to observe the teaching of participating teachers before sub-study 1 and 2. Because of the timing of the study and school schedules, the first point of contact with the teachers was during the TPD workshop. Data about the initial preferred practices of teachers was thus collected through questionnaire and interviews rather than through lesson observation.

4. The claims for teachers' learning and changes in practice have been made from observations by the researcher and the self-reports of the teachers. No standard instrument was used to assess the development of teachers' knowledge. However, in-depth analysis of teachers' discourse in workshops and their teaching provide evidence for their learning.

8.3. My journey from being a teacher to a researcher

When I joined the PhD program at HBCSE, I brought the perspective of being a teacher concerned about developing understanding of mathematics among students. The challenges that I had faced in teaching mathematics and the teaching approach taken in summer camps conducted for students by the mathematics education group in HBCSE inspired me to take up research in the field of mathematics education. My own school experience of learning mathematics and the difficulties and challenges faced in understanding mathematics has guided me to explore, together with other teachers, ideas and methods in this study that can make mathematics accessible to students. In the course of the research study, I was able to integrate the perspective of the teacher by appreciating the kind of

challenges faced by a teacher in implementing the intended changes. The interactions with other researchers and teacher educators and close study and analysis of their practice provided an opportunity to understand the aspects that contribute towards teachers' growth. Further, interactions with the teachers in the workshops as well as collaborating with them in their classrooms for teaching challenging topics helped in developing an appreciation of the aspects of practice and knowledge that contributes towards development of students' understanding. As a result of being engaged in these studies, I got an opportunity for growth in my own knowledge, and beliefs and practice about mathematics and its teaching. As a person not having a Bachelor's degree in mathematics, it was challenging at times but it was also an opportunity to learn while being actively engaged in the practice of studying the teaching of mathematics. I believe that my background helped me in being sensitive to the difficulties faced by the students and teachers.

References

Aguire, J. and Speer, N. (2000). Examining the relationship between beliefs and goals in teacher practice. *Journal of Mathematical Behaviour*, 18(3), 327-356.

Askew, M., Brown, M., Rhodes, V., Johnson, D., & Wiliam, D. (1997). *Effective teachers of numeracy*. London: Kings College.

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. *Teaching as the learning profession: Handbook of policy and practice*, 1, 3-22.Chicago

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide?. *American Educator,* Fall, 14-46.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content Knowledge for Teaching What Makes It Special?. *Journal of teacher education*, 59(5), 389-407.

Bajaj, R. & Kumar, R. S. (2012) A teaching learning sequence for integers based on real life context: A dream mall for children. Kharatmal, M. Kanhere, A. & Subramaniam, K. (Eds.). In *Proceedings of National conference on Mathematics Education* (pp.86-89). Mumbai: HBCSE.

Batra, P. (2005).Voice and Agency of Teachers: The missing link in the National. Curriculum Framework 2005, *Economic and Political Weekly*, 40(36),4347-4356.

Bishop, A. J. (1999). Mathematics teaching and values education—An intersection in need of research. *ZDM*, 31(1), 1-4.

Brodie, K. (2013). The power of professional learning communities. *Education as change*, 17(1), 5-18.

Cerbin, W., & Kopp, B. (2006). Lesson study as a model for building pedagogical knowledge and improving teaching. *International journal of teaching and learning in higher education*, 18(3), 250-257.

Clark, Moss, Goering, Herter, Lamar, Leonard, Robbins, Russell, Templin & Wascha (1996). Collaboration as dialogue: Teacher and researcher engaged in conversation and professional development. *American education research journal, 33*(1), 193-232.

Clarke, D. (1994). Ten key priciples from research on the professional development of mathematics teachers. In D. B. Aichele & A. F. Coxford (Eds.) *Professional development for teachers of mathematics.* (pp. 37-48). Reston, VA: National council of teachers of mathematics.

Cobb, P., Wood, T., & Yackel, E. (1990). Chapter 9: Classrooms as Learning Environments for Teachers and Researchers. *Journal for Research in Mathematics Education*. Monograph, 4, 125-210.

Cochran-Smith, M., & Lytle, S. L. (1999). Relationships of knowledge and practice: Teacher learning in communities. *Review of research in education*, 24, 249-305.

Cohen, D. K., & Ball, D. L. (1990). Relations between policy and practice: A commentary. *Educational Evaluation and Policy Analysis*, 12(3), 331-338.

Crespo, S. (2000). Seeing more than right and wrong answers: Prospective teachers' interpretations of students' mathematical work. *Journal of Mathematics Teacher Education*, 3(2), 155-181.

Da Ponte, J. P., & Chapman, O. (2006). Mathematics teachers' knowledge and practices. *Handbook of research on the psychology of mathematics education: Past, present and future*, 461-494.

Day, C. (1999). *Developing teachers: The challenges of lifelong learning*. Routledge.

diSessa, A. A. (2002). Students' criteria for representational adequacy. *Symbolizing, modeling and tool use in mathematics education*, 105-129.

Edwards, T. G., & Hensien, S. M. (1999). Changing instructional practice through action research. *Journal of Mathematics Teacher Education*, 2(2), 187-206.

Empson, S. B., & Jacobs, V. R. (2008). Learning to listen to children's mathematics. *Tools and processes in mathematics teacher education*, 257-281.

Erickson, G., Minnes Brandes, G., Mitchell, I., & Mitchell, J. (2005). Collaborative teacher learning: Findings from two professional development projects. *Teaching and Teacher Education*, 21(7), 787-798.

Even, R., & Ball, D. L. (Eds.). (2009). *The professional education and the development of teachers of mathematics*. New York: Springer.

Farmer, J. D., Gerretson, H., & Lassak, M. (2003). What teachers take from professional development: Cases and implications. *Journal of Mathematics Teacher Education*, 6(4), 331-360.

Fernandes, D., & Vale. I. (1994). Two young teachers' concepts and practice, Beliefs and Teaching Practice about problem solving. In J. P. Ponte & J. F. Matos (Eds.), *Proceedings of the 18th PME International Conference*,2, 328-335.

Goldin, G., Rösken, B., & Törner, G.(2009). Beliefs–No longer a hidden variable in math ematical teaching and learning processes. In J. Maaß, & W. Schlöglmann (Ed.), *Beliefs and Attitudes in Mathematics Education: New Research Results,* 1-18.

Graven, M. (2003). Teacher learning as changing meaning, practice, community, identity and confidence: The story of Ivan. *For the Learning of Mathematics*, 23(2), 28-36.

Graven, M. (2004). Investigating mathematics teacher learning within an in-service community of practice: The centrality of confidence. *Educational studies in mathematics*, 57(2), 177-211.

Grossman, P., Wineburg, S., & Woolworth, S. (2001). Toward a theory of teacher community. *The Teachers College Record*, 103(6), 942-1012.

Guskey, T. R. (2000). *Evaluating professional development*. Thousand Oaks, CA: Corwin Press.

Hannula, M. S., Liljedahl, P., Kaasila, R., & Rösken, B. (2007). Researching relief of mathematics anxiety among pre-service elementary school teachers. In J.-H. Woo, H.-C. Lew, K.-S. Park, & D.Y. Seo (Eds.), *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 153-157). Seoul, Korea: PME.

Jaworski, B. (2007). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education*, 9(2), 187-211.

Jaworski, B. (2008). Building and sustaining inquiry communities in mathematics teaching development: Teachers and didacticians in collaboration. In K. Krainer & T. Wood (Eds.), *International handbook of mathematics teacher education* (Vol. 3, pp. 335–361). Rotterdam: Sense Publishers.

Jaworski, B., Goodchild, S., Eriksen, S., & Daland, E. (2011). Mediating mathematics teaching development and pupils' mathematics learning: the life cycle of a task. *Constructing Knowledge for Teaching Secondary Mathematics*, 143-188.

Kagan, D. M. (1992). Professional growth among preservice and beginning teachers. *Review of educational research*, 62(2), 129-169.

Katz, S., & Earl, L. (2010). Learning about networked learning communities. *School Effectiveness and School Improvement*, 21(1), 27-51.

Kazemi, E., & Franke, M. L. (2004). Teacher learning in mathematics: Using student work to promote collective inquiry. *Journal of Mathematics Teacher Education*, 7(3), 203-235.

Kazemi, E., & Hubbard, A. (2008). New Directions for the Design and Study of Professional Development Attending to the Coevolution of Teachers' Participation Across Contexts. *Journal of Teacher Education*, 59(5), 428-441.

Kieren, T. E. (1976). On the mathematical, cognitive and instructional. In *Number and measurement*. Papers from a research workshop (Vol. 7418491, p. 101).

Kieren, T. E. (1988). Personal knowledge of rational numbers: Its intuitive and formal development. *Number concepts and operations in the middle grades*, 2, 162-181.

Kieren, T. E. (1993). Rational and fractional numbers: From quotient fields to recursive understanding. *Rational numbers: An integration of research*, 49-84. Chicago

Kumar, R. S., Subramaniam, K. (2016). Constraints and affordances in bringing about shifts in practice towards developing reasoning in mathematics: Case study. Kaur, B., Kwon, O. N., & Leong, Y. H. (eds.) *Professional Development of Mathematics Teachers. An Asian Perspective.* DOI 10.1007/978-981-10-2598-3_10

Kumar, R. S., & Subramaniam, K. (2015). From ''Following'' to Going Beyond the Textbook: Inservice Indian Mathematics Teachers'' Professional Development for Teaching Integers. *Australian Journal of Teacher Education*, *40*(12). http://dx.doi.org/10.14221/ajte.2015v40n12.7

Kumar, R. S., Subramaniam, K. & Naik, S. (2015a). Professional development of in-service teachers in India. Sriraman, B. , Cai, J., Lee, Kyeong-Hwa, Fan, L., Shimizu, Y., Lim, Chap Sam, & Subramaniam, K. (Eds.). F*irst sourcebook on Asian research in*

mathematics Education. Information Age publishers.

Kumar, R. S., Subramaniam, K. & Naik, S. (2015b). Teachers′ construction of meanings of signed quantities and integer operation. *Journal of Mathematics Teacher Education.* (pp: 1-34). Springer: Netherlands.

Kumar, R, S. & Subramaniam, K. (2013). Elementary Teacher''s belief about teaching of mathematics, (ed.) G. Nagarjuna, A. Jamakhandi & E. M. Sam. *In proceedings of Episteme- 5 conference to be held at HBCSE, Mumbai*. Goa: Common Teal Publishing.

Kumar, R. S., Subramaniam, K. & Naik, S. (2013). Professional development of in-service teachers in India. Sriraman, B. , Cai, J., Lee, Kyeong-Hwa, Fan, L., Shimizu, Y., Lim, Chap Sam, & Subramaniam, K. (Eds.). *Abstracts of the first sourcebook on Asian research in mathematics Education.* (pp. 207-211). Information Age publishers.

Kumar, R. S. & Subramaniam, K. (2012a). Interaction between belief and pedagogical content knowledge of teachers while discussing use of algorithms. In Tso, T. Y. (Ed). *Proceedings of the 36th conference of the International group for the Psychology of Mathematics Education*, Vol. 1, pp. 246. Taipei, Taiwan: PME

Kumar, R. S. & Subramaniam, K. (2012b). One teachers struggle to teach equivalent fractions with meaning making. In Tso, T. Y. (Ed). *Proceedings of the 36th conference of the International group for the Psychology of Mathematics Education.* Vol. 4, pp. 290. Taipei, Taiwan: PME.

Kumar, R. S. & Subramaniam, K. (2012c). Understanding teachers' concerns and negotiating goals for teaching: insights from collaborative lesson planning. In proceedings of 12th International Congress of Mathematical education, pp.5157-5166, Seoul, Korea: ICME.

Kumar, R. S. Dewan, H., & Subramaniam, K. (2012). The preparation and professional development of mathematics teachers. In Ramanujam, R. & Subramaniam K. *Mathematics Education in India: Status and Outlook*. pp. 151-182. HBCSE, Mumbai.

Lewis, C. C., Perry, R. R., & Hurd, J. (2009). Improving mathematics instruction through lesson study: A theoretical model and North American case. *Journal of Mathematics Teacher Education*, 12(4), 285-304.

Little, J. W. (1993). Teachers' professional development in a climate of educational reform.

Educational evaluation and policy analysis, 15(2), 129-151.

Ma, L. (1999). Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in china and the united states, *Studies in mathematical thinking and learning*, Lawrence Erlbaum associates: Mahwah, NJ.

Matos, J. F., Powell, A., Sztajn, P., Ejersbø, L., Hovermill, J., & Matos, J. F. (2009). Mathematics teachers' professional development: processes of learning in and from practice. *The professional education and development of teachers of mathematics*, 167-183.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: A sourcebook*. Beverly Hills: Sage Publications.

National Council of Educational Research and Training (2005). *National curriculum framework*. New Delhi: NCERT.

National Council of Educational Research and Training. (2006). *National focus group on teaching of mathematics report*. New Delhi: NCERT.

Nisbet, S., Warren, E., & Cooper, T. (2003). Collaboration and sharing as crucial elements of professional development. *Collaboration in teacher education: Examples from the context of mathematics education*, 23-40.

Olson, J.K. [1985] Changing our ideas about change. *Canadian Journal of Education*, 10, pp.294-307

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research*, 62(3), 307-332.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257-315). Reston, VA: National Council of Teachers of Mathematics.

Potari, D., Sakonidis, H., Chatzigoula, R., & Manaridis, A. (2010). Teachers' and researchers' collaboration in analysing mathematics teaching: A context for professional reflection and development. *Journal of Mathematics Teacher Education*, 13(6), 473-485.

Raymond, A. M., & Leinenbach, M. (2000). Collaborative action research on the learning and teaching of algebra: a story of one mathematics teacher's development. *Educational*

Studies in Mathematics, 41(3), 283-307.

Roesken, B. (2011). *Hidden dimensions in the professional development of mathematics teachers- Inservice education for and with teachers*. Rotterdam: Sense Publishers.

Sahin, A., & Kulm, G. (2008). Sixth grade mathematics teachers' intentions and use of probing, guiding, and factual questions. *Journal of mathematics teacher education*, 11(3), 221-241.Chicago

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. *Handbook of research on mathematics teaching and learning*, 334-370.

Schoenfeld, A. H. (1999). Models of the teaching process. *The Journal of Mathematical Behavior*, 18(3), 243-261.Chicago

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher,* Feb. 1986: 4-14. (AERA Presidential Address).

Speer, N. (2005). Issues of methods and theory in the study of mathematics teachers' professed and attributed beliefs, *Educational studies in mathematics*, 58, (pp.361-391), Springer.

Stephan, M., & Akyuz, D. (2012). A proposed instructional theory for integer addition and subtraction. *Journal for Research in Mathematics Education*, 43(4), 428-464.Chicago

Stoll, L., Bolam, R., McMahon, A., Wallace, M., & Thomas, S. (2006). Professional learning communities: A review of the literature. *Journal of Educational Change*, 7(4), 221-258.

Swan, M. (2006). *Collaborative learning in mathematics: A challenge to our beliefs and practices*. Liecester, UK :National institute of adult continuing education.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New york: Macmillian.

Thompson, A. G., Thompson, P. W., Boyd, B. A., (1994). Calculational and conceptual orientations in teaching mathematics. In In DB Aichele & AF oxford (Eds.), *Professional Development for Teachers of Mathematics*: 1994 Yearbook. Reston, VA: National Council

of Teachers of Mathematics. Chicago

Thompson, P. W., & Dreyfus, T. (1988). Integers as transformations. *Journal for Research in Mathematics Education*, 115-133.

Vergnaud, G. (1982). A classification of cognitive tasks and operations of thought involved in addition and subtraction problems. *Addition and subtraction: A cognitive perspective*, 39-59.

Vescio, V., Ross, D., & Adams, A. (2008). A review of research on the impact of professional learning communities on teaching practice and student learning. *Teaching and Teacher Education*, 24(1), 80-91.

Vlassis, J. (2004). Making sense of the minus sign or becoming flexible in 'negativity'. *Learning and instruction*, 14(5), 469-484.

Vlassis, J. (2008). The role of mathematical symbols in the development of number conceptualization: The case of the minus sign. *Philosophical Psychology*, 21(4), 555-570.

Watson, A. & Mason, J. (2005). *Mathematics as a Constructive Activity: the role of learner-generated examples*, Mahwah USA: Erlbaum.

Wilson, M. S. & Cooney, T. (2002). Mathematics teacher change and development. In G. C. Leder, E. Pehkonen, & G. Torner (Eds.), *Beliefs: A hidden variable in mathematic education* (pp. 127-147). Dordrecht, The netherlands: Kluwer.